Laboratory No. 06:

Ball and Beam Experiment with a Full Order Estimator

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BYU-Idaho CompE 470

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I.   OBJECTIVES

The objective for this laboratory is to design, implement, and document Full Order Estimator for the ball-and-beam experiment.

II. PRELIMINARY

Using the full state feedback controller designed in Lab #5, design a full state estimator and replace the state used in Lab #5 with the estimated state. What are the estimator gain values and associate pole values? How are the estimator gains calculated? What pole values were used in the design and why? Simulate the design, showing the convergence of the estimated states to the modeled plant states.

A. Formulas and Overall Descriptions Used

Most of this laboratory's calculations were completed within MatLab 2007b, so some of the following formulas have been simplified to commands based within that program (lines appearing `as such` are MatLab commands).

$$\begin{bmatrix} l_1 \\ l_2 \end{bmatrix} = \mathbf{L} = \texttt{acker}(A_c\text{'},C_c\text{'},P)\text{'}$$

$$\dot{x}_1 = u$$
$$\dot{x}_2 = x_1$$

$$y = \begin{bmatrix} 0 & \alpha \end{bmatrix} \mathbf{x}$$
$$x_{TB} = y = \alpha x_2$$

$$u = -KX + \bar{N}r$$
$$u = -k_1 x_1 - k_2 x_2 + \frac{1}{\alpha} k_2 r$$

$$\dot{\hat{x}} = F\hat{X} + GU + \mathbf{L}\tilde{y}$$
$$\hat{y} = \begin{bmatrix} 0 & \alpha \end{bmatrix} \hat{x}$$

B. Overall Diagrams

This section will consist of the diagrams and schematics used for this laboratory. As a note, only the full order estimator with the full state feedback controller will be shown below since the overall system has been detailed in previous labs (specifically, Lab #4).
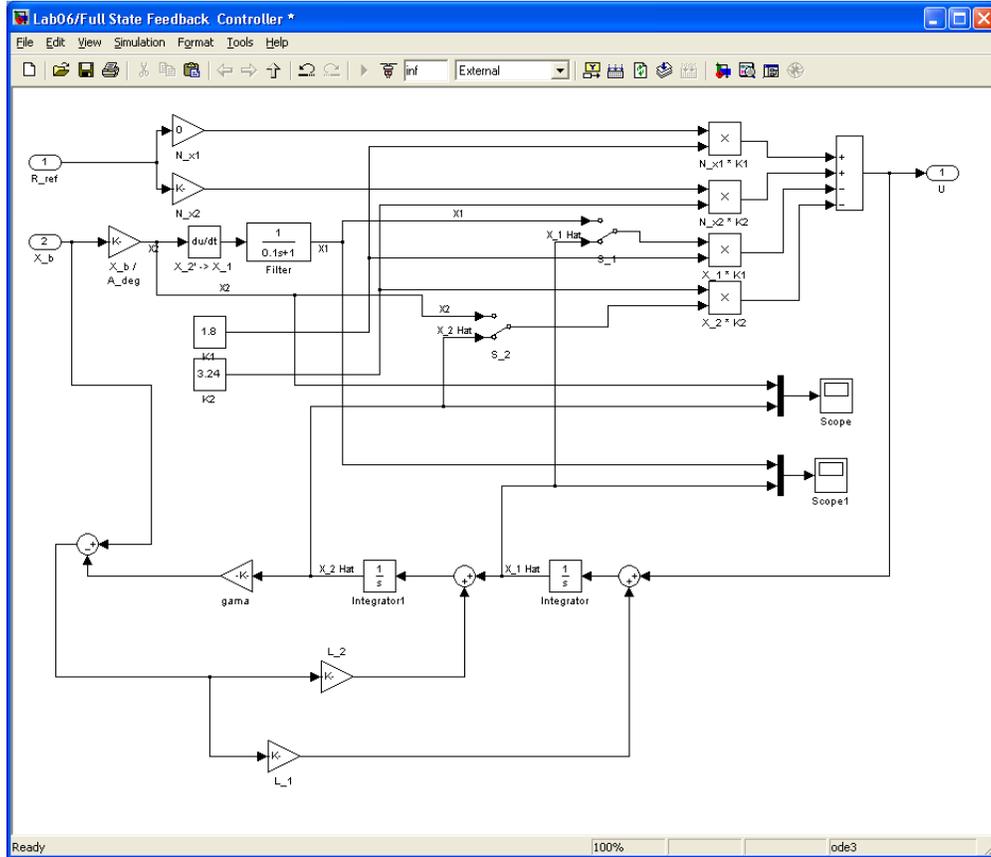

FIG. 1: Overall view of the full order estimator with full state feedback control above.

III. PROCEDURES

A. Equipment

- Ball and Beam apparatus (pre-assembled)
- MatLab 2007b Software
- National Instruments hardware to connect to MatLab 2007b

B. Schematics

There were no circuit schematics used in this laboratory. However, there were block diagrams used. Please refer to Figure 1 for an overall view of the system within MatLab.

C. Circuit Results

There are no circuit results to be presented.

D. Explanation and Data

A quick recap of the control canonical form of our transfer function for this laboratory will reveal that . . .

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0.925 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \end{bmatrix}$$

Then after choosing values for our pole locations based on quick convergence and little noise . . .

$$P = \begin{bmatrix} -60 & -50 \end{bmatrix}$$

Plugging these values into MatLab and performing the command `acker(A',B',P)'` reveals $\mathbf{L}$ . . .

$$\mathbf{L} = \begin{bmatrix} 3243.2 \\ 118.9 \end{bmatrix}$$

E. Simulations

Below are screengrabs from the simulations of this laboratory. Since most of the simulation information is in the section above (Explanation and Data), I will not be expanding upon these graphs.
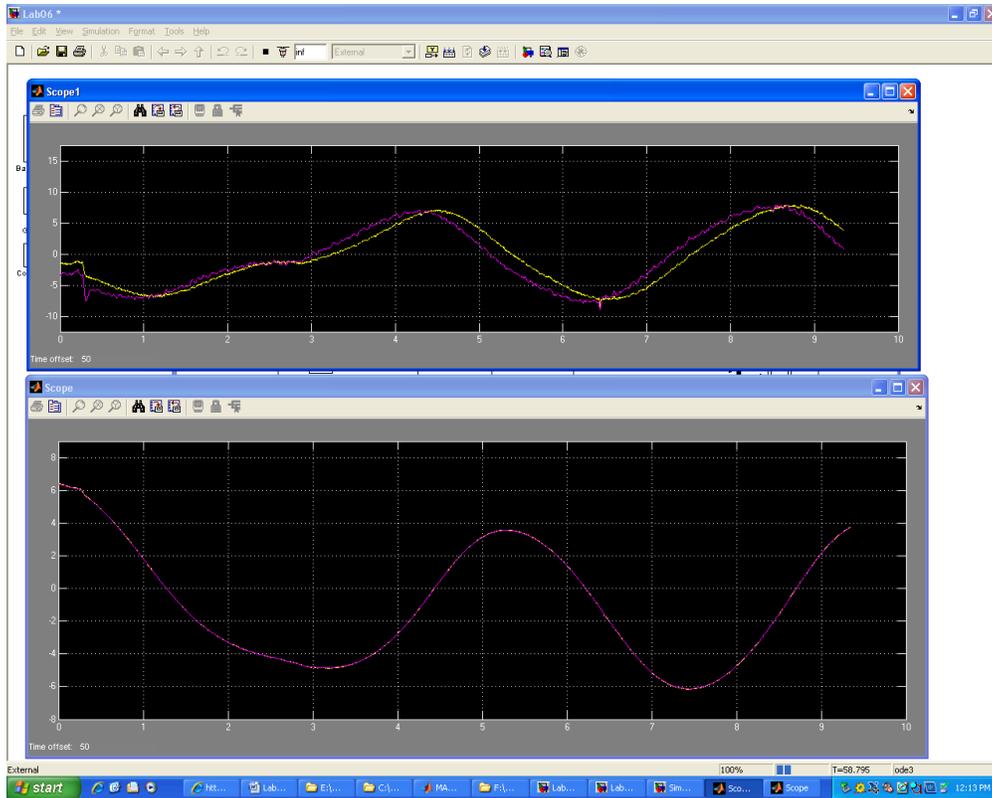
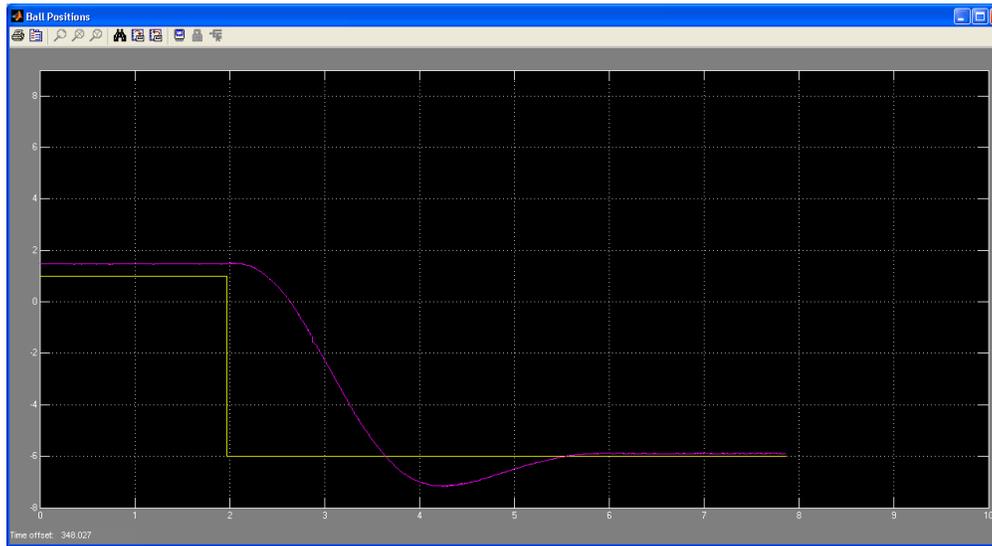FIG. 2: Convergence of the estimator when comparing x_n and $\hat{x}_n$

FIG. 3: Output of system with only estimator and a gain of $5 * k$.

6

IV. CONCLUSION

This laboratory taught students how to design, implement, and document full order estimator control for the ball-and-beam experiment.

The full order estimator was actually quite easy to implement into the system and it gave surprisingly good results. As a group, we did hit a few stumble blocks along the way. Specifically, Brother Smith had given us a α value to be used which was supposed to be more accurate ($\alpha = 0.17$). After first going through the calculations for our system with this value, we quickly realized that it wasn't working very well. We went back and substituted $\alpha = 0.925$ (a given in Lab 05), the system worked really well. We concluded that we were going to stick with this α value instead.

After putting the full order estimator in place, we hooked up some switches so that we could switch between the full order estimator and the full state feedback systems. This allowed us to quickly compare the two. Both were very similar in performance, but I would have to say that the winner, if there were one, is the estimator because of its ease of use and implementation.

There were questions posed within the lab which students needed to answer. I answered them in the lab write-up, but also wanted to answer them here. So . . .

Q What are your estimator gain values and associate pole values?

A $\mathbf{L} = 3243.2$ and 119.9; $\mathbf{P} = -60$ and -50

Q How did you calculate your estimator gains?

A By using the MatLab command `acker(A',B',P)'`.

Q Why did you select the poles values used in your design?

A We selected them really by guess, but as an educated guess, we chose a smaller value so that we had less noise (as we learned in class). It couldn't be too small, or there wouldn't be a quick convergence (Figure 2).

Overall, I liked this lab. It was interesting to see how quickly the numbers fell out as well as how accurate the system was when we finished it (and the size of this lab write-up shows how simple the calculations were). It was a little difficult to get the diagram right at first, but once we thought things through, we ended up with a very nice looking response from the system (as seen in Figure 3).